Nonlinear Lamination Stacks Studied with Harmonic Balance FEM combined with Homogenization approach

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Harmonic Balance Finite Element method combined with homogenization method is used to model lamination stack. The Harmonic Balance gives directly the steady-state solution and the homogenization method reduces the number of unknowns. The numerical model takes into account the nonlinear magnetic behavior and the electric conductivity. The results of the proposed method are compared with those obtained from a classic approach.

***Index Terms*—Finite element method, fixed point method, Harmonic Balance method, homogenization method, lamination stacks.**

I. Introduction

Iron cores in electrical devices are usually made of lamination stack. Simulating such devices and modeling eddy currents in each separated lamination using finite element method (FEM) is computationally expensive due to the spatial discretization on the scale of individual lamination. Homogenization methods offer a good approximation, by replacing the stacked region with an equivalent homogenous medium. Indeed, it can greatly reduce the size of the mesh. Homogenization methods are usually combined with a time stepping finite element method (TS-FEM) [1][2]. In most electrical engineering applications, the steady-state solution is usually sought. With a time stepping scheme, a lot of periods could be required in order to obtain the steady-state solution. Thus, the computation time is expensive especially in the case of a large transient state. To tackle this issue, the harmonic balance approach (HB) combined with the FEM (HB-FEM) can be investigated in order to compute directly the steady of the solution. The HB is based on the expansion of the solution as a Fourier series [3][4].

The HB was introduced in the late 1980's to analyze electromagnetic field problems [5], where the HB method has been combined with FEM. In [6] we have proposed a HB-FEM applied to lamination stack.

In this communication, the HB-FEM combined with a homogenization method (HB-FEM-H) is developed to simulate a laminated iron core. The steel sheets are modelled by a nonlinear magnetic behaviour and a linear isotropic electric conductivity. The results from the HB-FEM-H are compared with those obtained from the TS-FEM, TS-FEM homogenization (TS-FEM-H) and the HB-FEM.

II. Non-linear magneto-quasistatic problem

We consider a lamination of thickness $d(-d/2\leq y\leq -d/2)$ that carries the magnetic flux density $\boldsymbol{B}$ and magnetic field $\boldsymbol{H}$ along the z axis. Consequently, the axis x is the direction of the eddy-current loops generated by the variation of $\boldsymbol{B}$ with the approximation of neglecting the edge effect (Fig. 1).

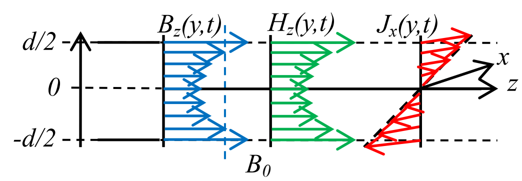

Fig. 1. Variation of $\boldsymbol{B}$, $\boldsymbol{H}$ and $\boldsymbol{J}$ throughout lamination thickness.

To study a lamination stack, a magneto-quasistatic problem can be solved using the vector potential $\boldsymbol{A}$. In this case, the magnetic flux density $\boldsymbol{B}$ and the electric field $\boldsymbol{E}$ can be written such as $\boldsymbol{B}=\boldsymbol{curlA}+\boldsymbol{N}\Phi$ and $\boldsymbol{E}=-\partial_t\boldsymbol{A}-\partial_t\boldsymbol{K}\Phi$ with Φ the imposed magnetic flux and $\boldsymbol{N}\Phi$ and $\boldsymbol{K}\Phi$ the source fields [7] such as $\boldsymbol{curlK}=\boldsymbol{N}$. Using a the fixed point approach, the nonlinear behavior of the ferromagnetic material can be expressed by $\boldsymbol{H}=\nu_{fp}\boldsymbol{B}+\boldsymbol{H}_{fp}(\boldsymbol{B})$ with ν_{fp} a constant and $\boldsymbol{H}_{fp}(\boldsymbol{B})$ a virtual magnetization [8].

Considering a heterogeneous lamination stack, the equation to be solved is

$$\begin{aligned}&\boldsymbol{curl}(\nu_{fp}\boldsymbol{curlA})+\sigma\partial_t\boldsymbol{A}=-\boldsymbol{curl}(\nu_{fp}\boldsymbol{curl\,K}\Phi)\\&-\sigma\partial_t\boldsymbol{K}\Phi-\boldsymbol{curl}(\boldsymbol{H}_{fp}(\boldsymbol{B}))\end{aligned}\quad(1)$$

The homogenization method used is based on a spatial polynomial expansion of the magnetic flux density $B_z(y,t)=\sum_{i=0}^{n}\alpha_i(y)B_i(t)$, where the polynomial basis functions $\alpha_1(y)=1$, $\alpha_2(y)=-(1/2)+6(y/d)^2,\ldots$, are orthogonals [1][2]. The homogenized formulation can be obtained combining the vector expansion of $B_z(y,t)$ for $n=2$ and the equation (1):

$$\begin{aligned}&\boldsymbol{curl}(\nu_{fp}\boldsymbol{curlA_0})+\frac{\sigma d^2}{12}\partial_t\boldsymbol{curlA_0}-\frac{\sigma d^2}{60}\partial_t\boldsymbol{B_2}=\\&\boldsymbol{curl}(\nu_{fp}\boldsymbol{curl\,K}\Phi)-(\nu(\boldsymbol{B})-\nu_{fp})\boldsymbol{curlA_0}\\&\nu_{fp}\boldsymbol{B_2}-\frac{\sigma d^2}{60}\partial_t\boldsymbol{curlA_0}+\frac{\sigma d^2}{120}\partial_t\boldsymbol{B_2}=-(\nu(\boldsymbol{B})-\nu_{fp})\boldsymbol{B_2}\end{aligned}\quad(2)$$

with $\boldsymbol{B_0}=curl\boldsymbol{A_0}$ the averaged value of the magnetic flux density. Using a spatial semi-discretisation of $\boldsymbol{A}$, $\boldsymbol{A_0}$ and $\boldsymbol{B_2}$, the system of ordinary differential equations to solve respectively for the equation (1) and (2) can be written in a condensed form:

$$\boldsymbol{N}\mathbf{x}(t)+\boldsymbol{M}\frac{d\mathbf{x}(t)}{dt}=\boldsymbol{F}(t)-\boldsymbol{M}_{fp}(\mathbf{x}(t))\quad(3)$$

Where the vector $\mathbf{x}(t)$ contains the discrete values of $\boldsymbol{A}$ for the equation (1) and $\boldsymbol{A_0}$, $\boldsymbol{B_2}$ for the equation (2). $\boldsymbol{N}$ and $\boldsymbol{M}$ are square matrices, $\boldsymbol{F}(t)$ is the source vector and $\boldsymbol{M_{fp}}(\mathbf{x}(t))$ the vector composed of entries depending on the solution $\mathbf{x}(t)$. Applying the HB-FEM, the vector $\mathbf{x}(t)$ is expanded as a complex Fourier series with N harmonics

$$\mathbf{x}(t) \cong Re(\boldsymbol{X_0}+\sum_{k=1}^{N} \underline{\boldsymbol{X}_k} e^{j\omega kt}) \qquad (4)$$

With ω the angular frequency, $\boldsymbol{X_0}$ the DC-component and $\underline{\boldsymbol{X}_k}$ the complex magnitude at the k^{th} angular frequency. Then, $\mathbf{x}(t)$ in (3) is replaced by (4). The numerical model is obtained by multiplying (4) by a set of test functions $e^{-j\omega kt}$, $k=1,...,N$ and integrating it over a period T_p. Then, the system to be solved is composed of N equations

$$(\boldsymbol{N}+j\omega k\boldsymbol{M})\,\underline{\boldsymbol{X}_k} = \frac{1}{T_p}\int \left(\boldsymbol{F}(t) - \boldsymbol{M_{fp}}(\mathbf{x}(t))\right) e^{-j\omega kt}\,dt \qquad (5)$$

This nonlinear model is solved by using the fixed point approach.

III. Application

The HB-FEM-H is applied to a 2D model of two rectangular steel sheets. A sinusoidal magnetic flux is imposed perpendicularly to the lamination stack with a frequency of $f=500Hz$. In Fig. 2, the steady-state of the magnetic field from TS-FEM (reference) with and without homogenization is compared to the one calculated by the HB-FEM with and without homogenization. The four curves show a good agreement. In Fig. 3, the spectrums of the magnetic fields of the different formulations are compared to the reference. It brings out that the fundamental (*500Hz*), the third *(1500Hz)* and the fifth (2*500Hz*) harmonic are dominant. The amplitudes at *500Hz*, 1*500Hz* and 2*500Hz* are very close for the four methods. The error of the HB-FEM, TS-FEM-H and HB-FEM-H and are obtained using the L^2 relative error norm for the steady-state of the magnetic field and are respectively, 0.1874, 0.3213 and 0.3975. The relative error of the HB-FEM is due to the approximation of complex Fourier series. While, the relative error of the TS-FEM-H is because the homogenization method which does not take into account the edge effect. Finally, the error of the HB-FEM-H is a combinaison of two reasons cited above.

In terms of computation times, the speed up of the HB-FEM-H, HB-FEM and TS-FEM-H are respectively 20, 4 and 2.

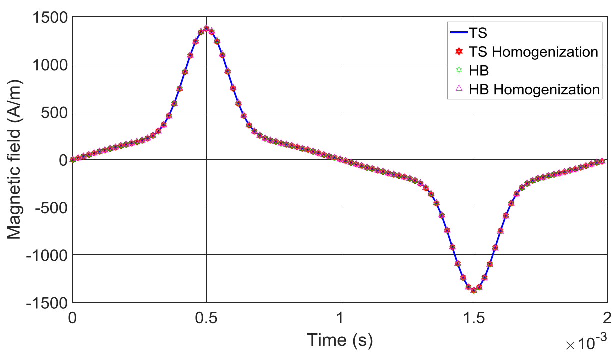

Fig. 2. Comparison of the magnetic field at $f = 500Hz$.

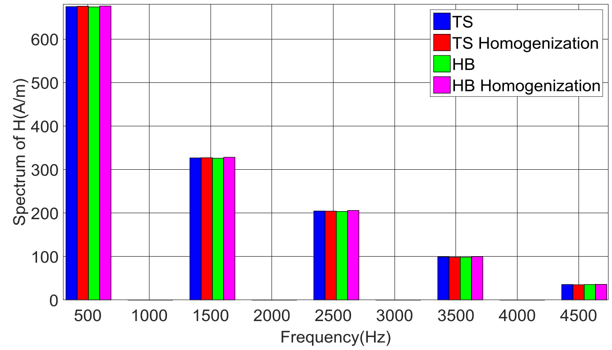

Fig. 3. Comparison of the spectrum of the magnetic field at $f = 500Hz$.

IV. Conclusion

The HB-FEM-H combined with the fixed point method is used to solve a nonlinear magneto-quasistatic problem applied to lamination stack. With the studied example, the proposed method gives results with a good accuracy compared with a classic approach with a significant speed up of 20. The HB-FEM-H has a relative error of 0.3975 and this because the homogenization method used does not take into account the edge effect in each sheet. To dicrease this error it is possible to take into account the edge effect by coupling the HB-FEM with the HB-FEM-H. The HB-FEM can be used near the edges of each sheet and the rest of the domain modeled by using the HB-FEM-H. The method proposed in [9] enables to couple the HB-FEM and a homogenized formulation in order to take into account the edge effect in lamination stack in linear case. This approach will be extended with HB-FEM and HB-FEM-H.

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